Densities of the Ternary Systems $Y(NO_3)_3 + Ce(NO_3)_3 + H_2O$, $Y(NO_3)_3 + Nd(NO_3)_3 + H_2O$, and $Ce(NO_3)_3 + Nd(NO_3)_3 + H_2O$ and Their Binary Subsystems at Different Temperatures

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Densities were measured for the ternary systems $Y(NO_3)_3 + Ce(NO_3)_3 + H_2O$, $Y(NO_3)_3 + Nd(NO_3)_3 + H_2O$, and $Ce(NO_3)_3 + Nd(NO_3)_3 + H_2O$ and their binary subsystems at (293.15, 298.15, and 308.15) K. The results were used to test the applicability of simple equations for the density of the mixed solutions. The predictions are in good agreement with measured values, implying that the densities of the examined electrolyte solutions can be well predicted from those of their constituent binary solutions by the simple equations.

Introduction

The thermodynamic and transport properties of mixed aqueous electrolyte solutions play an important role in a variety of fields such as chemistry and chemical engineering, separation processes, wastewater treatment, pollution control, and oil recovery. Up to now, extensive data have been reported in the literature for the thermodynamic and transport properties of binary aqueous electrolyte solutions, but relatively few measurements have been made on multicomponent electrolyte solutions, especially aqueous solutions of (1:3 + 1:3) electrolyte mixtures. At the same time, one of the objectives of the theory of electrolyte solutions is to calculate various properties of mixed electrolyte solutions in terms of the properties of binary solutions, and much effort has indeed been made in the literature to develop simple equations that can make full use of the available information on binary electrolyte solutions and provide sufficient accuracy to predict the properties of mixed solutions.[1−7] Up to now, such simple equations have been established for thermodynamic properties, including activity coefficients of either solute in the mixed solutions, volumetric properties, thermal properties, and surface tension.[1−7] There are several simple approaches for prediction of densities of mixed solutions, such as the rule of Young and Smith,[8] the rule of Patwardhan and Kumar,[6] and the semi-ideal solution theory.[1,2] These approaches could provide predictions for the thermodynamic properties of the mixed solutions in terms of the properties of their binary subsystems. Their accuracy has been tested by systematic comparisons with the experimental data for the mixed nonelectrolyte solutions,[9a] the mixed electrolyte solutions,[9b] and the mixed solutions of electrolytes and nonelectrolytes.[10] However, because the densities of aqueous solutions of (1:3 + 1:3) electrolyte mixtures are not available in the literature to our knowledge, tests were limited to aqueous solutions of (1:1 + 1:1), (1:1 + 1:2), and (1:1 + 1:3) electrolyte mixtures and limited to lower ionic strength. Therefore, in this study the densities were measured for the ternary systems $Y(NO_3)_3 + Ce(NO_3)_3 + H_2O$, $Y(NO_3)_3 + Nd(NO_3)_3 + H_2O$, and $Ce(NO_3)_3 + Nd(NO_3)_3 + H_2O$ and their binary subsystems $Y(NO_3)_3 + H_2O$, $Ce(NO_3)_3 + H_2O$, and $Nd(NO_3)_3 + H_2O$ at different temperatures and up to $I_{max} \leq 24.5$ mol·kg^{-1} (I is ionic strength). The results were used to check the predictability of the well-known approaches.

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Experimental Section

$Y(NO_3)_3 \cdot 6H_2O$ (99.99 %), $Ce(NO_3)_3 \cdot 6H_2O$ (99.99 %), and $Nd(NO_3)_3 \cdot xH_2O$ (> 99 %) supplied by Shanghai Aladdin Reagent Co., Ltd. were dissolved into double-distilled deionized water. The resulting rare earth nitrate solutions were adjusted to their equivalent concentrations with dilute HNO_3 solutions and then reheated and readjusted until stabilized.[11] The molalities of rare earth nitrate stock solutions were analyzed by both EDTA[12] and sulfate methods.[11] The stock solution concentrations were determined with an accuracy of ≤ 0.10 %.[12]

The experimental procedures are similar to those used in our previous study[9a,b] and are described briefly as follows. Dilute solutions were prepared by diluting a stock solution by mass. We prepared the ternary solutions by mixing the binary solutions. All solutions were prepared immediately before use, and the uncertainty was ± 5·10^{-5} mol·kg^{-1}. Densities of solutions were measured with a KEM oscillating-tube digital densimeter (DA-505) thermostatted to better than ± 0.01 K. The temperature in the measuring cell was monitored with a digital thermometer. The densimeter was calibrated with double-distilled water and dry air.[9,13−16] The densities of water at different temperatures were obtained from the literature.[17] The densities of dry air at different temperatures were taken from ref 18. In all the measured variables, the uncertainty in densities was ± 5·10^{-5}g·cm^{-3}.

Predictive Equations for Density of Mixed Electrolyte Solutions. In the following section, the variables with the superscript (*io*) together with the subscript M_iX_i were used to denote the quantities of component M_iX_i in the binary solution $M_iX_i + H_2O$ ($i = 1, 2, ..., n$) having the same water activity as that of a mixed solution, and those without the superscript (*io*) denote the corresponding quantities in the mixed solution.

The linear isopiestic relation[1−3,19] can be expressed as

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$$\sum_i \frac{m_{M_iX_i}}{m^{(io)}_{M_iX_i}} = 1 \qquad \left(a_w = \text{constant and } 0 \le \frac{m_{M_iX_i}}{m^{(io)}_{M_iX_i}} \le 1\right) \tag{1}$$

where $m_{M_iX_i}$ and $m^{(io)}_{M_iX_i}$ are the molalities of M_iX_i in the mixed aqueous solution $M_1X_1 + ... + M_iX_i + H_2O$ and its binary subsystems $M_iX_i + H_2O$ ($i = 1, 2, ..., n$) of equal water activity.

According to the semi-ideal solution theory, the density of a multicomponent electrolyte solution is related to those of its constituent binary solutions of equal water activity by[1,2]

$$\rho = \sum_i Y_{M_iX_i} / \sum_i (Y_{M_iX_i}/\rho^{(io)}_{M_iX_i}) \tag{2}$$

with $Y_{M_iX_i} = m_{M_iX_i}/m^{(io)}_{M_iX_i} + m_{M_iX_i}M_{M_iX_i}$, where m, ρ, and M denote molality, density, and molar mass. The density equation of Patwardhan and Kumar[6] can be expressed as

$$\rho = \sum_i Y_{M_iX_i} / \sum_i (Y_{M_iX_i}/\rho^{o,I}_{M_iX_i}) \tag{3}$$

with $Y_{M_iX_i} = y_{M_iX_i} + m_{M_iX_i}M_{M_iX_i}$, where $y_{M_iX_i}$ is ionic strength fraction and $\rho^{o,I}_{M_iX_i}$ is the density of the binary solutions having the same ionic strength fraction as that of the mixed solution.

Comparisons with the Experimental Data. The measured densities were used to test eqs 2 and 3, and the test procedure is briefly summarized as follows:

(1) Represent the measured densities of the binary solutions by the following polynomial equations

$$\rho^o_{M_iX_i(\text{calc})} = \sum_{l=0}^{N} A_l (m^o_{M_iX_i})^l \tag{4}$$

where $\rho^o_{M_iX_i(\text{calc})}$ and $m^o_{M_iX_i}$ denote the density and molality of the binary aqueous solution $M_iX_i + H_2O$ ($i = 1, 2, ..., n$). The optimum fit was obtained by variation of $l \le 5$ until the value of $\delta^o_{\rho,M_iX_i} = \sum_{j=1}^{N}(|\rho^o_{M_iX_i(\text{calc})} - \rho^o_{M_iX_i(\text{exp})}|/\rho^o_{M_iX_i(\text{exp})})/N$ is less than a few parts in 10^{-5}. The values of A_l and δ^o_{ρ,M_iX_i} obtained for the three binary solutions are shown in Table 1.

(2) Determine the compositions ($m^{(io)}_{M_iX_i}$) of the binary solutions having the same water activity as that of the mixed solution of given molalities $m_{M_iX_i}$ ($i = 1, 2, ..., n$) using the osmotic coefficients of M_iX_i ($i = 1, 2, ..., n$)[20−22] and eq 1. Note that the reported osmotic coefficient data[20−22] were represented by the polynomial equations $\varphi^o_{M_iX_i(\text{calc})} = \sum_{l=0}^{N} B_l (m^o_{M_iX_i})^{l/2}$. The values of B_l obtained for the binary solutions are shown in Table 1.

(3) Determine the compositions ($m^{o,I}_{M_iX_i}$) of the binary solutions having the same ionic strength as that of the mixed solution of given molalities $m_{M_iX_i}$ ($i = 1, 2, ..., n$).

(4) Insert the values of $\rho^{(io)}_{M_iX_i}$ and $\rho^{o,I}_{M_iX_i}$ calculated from eq 4 into eqs 2 and 3 to yield the predictions for the mixed solutions of given $m_{M_iX_i}$ ($i = 1, 2, ..., n$), which are then compared with the corresponding experimental data.

In this paper, the average relative differences between the predicted and measured densities (δ_ρ) over the entire experimental composition range of the mixed solution are defined by[9]

$$\delta_\rho = \sum_{i=1}^{N} |\delta_{\rho,i}|/N \tag{5}$$

with $\delta_{\rho,i} = (\rho_{i,(\text{calc})} - \rho_{i,(\text{exp})})/\rho_{i,(\text{exp})}$, where N is the number of experimental data.

Results and Discussion

Table 2 shows the measured densities of the binary solutions $Y(NO_3)_3 + H_2O$, $Ce(NO_3)_3 + H_2O$, and $Nd(NO_3)_3 + H_2O$ at different temperatures. Figure 1 shows the variations of the values of $\Delta\rho_{M_iX_i}$ ($= \rho^o_{M_iX_i(\text{calc})} - \rho^o_{M_iX_i(\text{exp})}$) with the molality in the binary solutions $Y(NO_3)_3 + H_2O$, $Ce(NO_3)_3 + H_2O$, and $Nd(NO_3)_3 + H_2O$ at 298.15 K, where $\rho^o_{M_iX_i(\text{exp})}$ is the density reported in the literature.[21−23] $\rho^o_{M_iX_i(\text{calc})}$ is the density calculated from eq 4 together with the parameters shown in Table 1. It is seen that the agreements are good. It should be noted that the stock solution concentrations were determined with an accuracy of ≤ 0.10 %,[12] which means that the density can be determined with an accuracy of ≤ 0.0004 g·cm^{-3}. The fitted parameters were extrapolated to calculate the densities for the binary solution $Y(NO_3)_3 + H_2O$ in the molality range ($0.23521 \le m^o_{M_iX_i} \le 0.41718$) mol·kg^{-1}. The calculated densities were then compared with the densities measured by Hakin et al.,[24] and the agreement is good, with $\delta_\rho = 1.8\cdot10^{-4}$.

Table 3 compares the predicted and measured densities for the ternary solutions $Y(NO_3)_3 + Ce(NO_3)_3 + H_2O$ at different temperatures. It is notable that the osmotic coefficients of its binary subsystems at 298.15 K are used to calculate the compositions ($m^{(io)}_{M_iX_i}$) of the binary solutions at (293.15 and 308.15) K. It is clear from the third to seventh columns of Table

Table 1. Parameters for the Binary Solutions $Y(NO_3)_3 + H_2O$, $Ce(NO_3)_3 + H_2O$, and $Nd(NO_3)_3 + H_2O$ at Different Temperatures

T/K	293.15	298.15	308.15		293.15	298.15	308.15
		$\rho^o_{Y(NO_3)_3+H_2O}$				$\rho^o_{Ce(NO_3)_3+H_2O}$	
A_0	0.997384	0.995502	0.992848	A_0	0.993492	0.994296	0.991069
A_1	0.229023	0.229400	0.225443	A_1	0.292139	0.283609	0.280844
A_2	−0.0263047	−0.0278444	−0.0263024	A_2	−0.0506119	−0.0424636	−0.0424357
10^3A_3	3.623389	4.365811	3.850967	10^3A_3	13.181371	9.345383	9.647246
10^4A_4	−5.751668	−7.222839	−6.270300	10^4A_4	−25.136345	−16.867863	−17.778089
10^5A_5	4.293403	5.350563	4.681642	10^5A_5	19.340987	12.774323	13.603820
δ^o_ρ	$3.0\cdot10^{-5}$	$4.9\cdot10^{-5}$	$4.2\cdot10^{-5}$	δ^o_ρ	$9.0\cdot10^{-6}$	$5.2\cdot10^{-5}$	$5.9\cdot10^{-5}$
		$\rho^o_{Nd(NO_3)_3+H_2O}$				$\varphi^{o,298.15}_{Y(NO_3)_3+H_2O}$	$\varphi^{o,298.15}_{Ce(NO_3)_3+H_2O}$	$\varphi^{o,298.15}_{Nd(NO_3)_3+H_2O}$
A_0	0.997386	0.995387	0.993223	B_0	0.816929	−3.299233	0.760181
A_1	0.281981	0.283925	0.277342	B_1	−0.343762	14.079971	−0.131740
A_2	−0.0335038	−0.0382564	−0.0335567	B_2	0.722643	−19.061765	0.132389
10^3A_3	5.050950	8.053080	5.875711	B_3	−0.164927	12.696013	0.207792
10^4A_4	−7.660146	−15.685357	−10.844769	B_4	0.010871	−4.046712	−0.102828
10^5A_5	5.254163	12.933185	8.940584	B_5	−0.000546	0.499920	0.014000
δ^o_ρ	$4.1\cdot10^{-5}$	$1.7\cdot10^{-5}$	$3.1\cdot10^{-5}$	δ^o_φ	$9.7\cdot10^{-4}$	$4.0\cdot10^{-3}$	$6.5\cdot10^{-4}$

Table 2. Densities of the Binary Solutions $Y(NO_3)_3 + H_2O$, $Ce(NO_3)_3 + H_2O$, and $Nd(NO_3)_3 + H_2O$ at Different Temperatures

(m_B)	$(\rho^o_{293.15})$	$(\rho^o_{298.15})/(g\cdot cm^{-3})$		$(\rho^o_{308.15})$
$(mol\cdot kg^{-1})$	$(g\cdot cm^{-3})$	exp	ref	$(g\cdot cm^{-3})$
		$Y(NO_3)_3$ (B) + H_2O (A)		
0.23521		1.0484[a]	1.0480[24]	
0.27747		1.0574	1.0571[24]	
0.32132		1.0666	1.0665[24]	
0.37064		1.0769	1.0769[24]	
0.41718		1.0865	1.0867[24]	
0.4989	1.1055	1.1035	1.1033[21]	1.0992
0.9965	1.2026	1.2002	1.1999[21]	1.1947
1.4972	1.2908	1.2878	1.2882[21]	1.2814
1.9820	1.3687	1.3654	1.3656[21]	1.3582
2.9475	1.5028	1.4989	1.4985[21]	1.4905
3.9452	1.6157	1.6114	1.6111[21]	1.6022
4.9575	1.7089	1.7043	1.7048[21]	1.6947
		$Ce(NO_3)_3$ (B) + H_2O (A)		
0.5112	1.1312	1.1293	1.1291[22]	1.1247
1.0313	1.2528	1.2502	1.2498[22]	1.2444
1.5546	1.3619	1.3588	1.3593[22]	1.3520
1.9969	1.4462	1.4431	1.4434[22]	1.4357
3.1308	1.6332	1.6291	1.6286[22]	1.6205
3.9676	1.7464	1.7424	1.7421[22]	1.7331
4.4616	1.8060	1.8018	1.8014[22]	1.7922
		$Nd(NO_3)_3$ (B) + H_2O (A)		
0.5220	1.1361	1.1342	1.1340[23]	1.1296
1.0213	1.2551	1.2525	1.2524[23]	1.2467
1.5554	1.3698	1.3667	1.3671[23]	1.3599
2.0934	1.4747	1.4711	1.4713[23]	1.4635
3.0857	1.6421	1.6378	1.6375[23]	1.6288
4.1242	1.7859	1.7811	1.7816[23]	1.7714
4.2972	1.8070	1.8027	1.8031[23]	1.7928

[a] Extrapolated values.

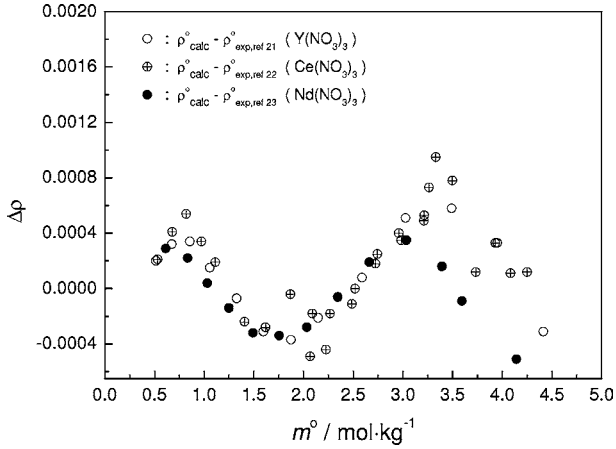

Figure 1. Variations of the $\Delta\rho$ values with the molalities for the binary solutions $Y(NO_3)_3 + H_2O$, $Ce(NO_3)_3 + H_2O$, and $Nd(NO_3)_3 + H_2O$ at 298.15 K.

3 that the agreements are good: the $\delta_\rho^{eq\ 2}$ values at (293.15, 298.15, and 308.15) K are $2.7\cdot10^{-4}$, $3.1\cdot10^{-4}$, and $3.4\cdot10^{-4}$, respectively. The values of $\delta_\rho^{eq\ 3}$ at (293.15, 298.15, and 308.15) K are $2.6\cdot10^{-4}$, $3.1\cdot10^{-4}$, and $3.6\cdot10^{-4}$, respectively.

Table 4 compares the predicted and measured densities for the ternary solutions $Y(NO_3)_3 + Nd(NO_3)_3 + H_2O$ at different temperatures. The $\delta_\rho^{eq\ 2}$ values at (293.15, 298.15, and 308.15) K are $4.4\cdot10^{-4}$, $4.8\cdot10^{-4}$, and $4.4\cdot10^{-4}$, respectively. The values of $\delta_\rho^{eq\ 3}$ at (293.15, 298.15, and 308.15) K are $4.2\cdot10^{-4}$, $5.0\cdot10^{-4}$, and $4.6\cdot10^{-4}$, respectively.

Table 5 compares the predicted and measured densities for the ternary solutions $Ce(NO_3)_3 + Nd(NO_3)_3 + H_2O$ at different temperatures. The $\delta_\rho^{eq\ 2}$ values at (293.15, 298.15, and 308.15) K are $2.1\cdot10^{-4}$, $2.9\cdot10^{-4}$, and $2.4\cdot10^{-4}$, respectively. The values

Table 3. Comparisons of Measured and Predicted Densities of the Ternary Systems $Y(NO_3)_3$ (B) + $Ce(NO_3)_3$ (C) + H_2O (A) at Different Temperatures and with $I_{max} \le 23.8$ mol·kg^{-1}

m_B		ρ $(g\cdot cm^{-3})$			$\Delta\rho$ $(g\cdot cm^{-3})$	
$(mol\cdot kg^{-1})$	m_C	exp	eq 2	eq 3	$\Delta_{eq}2$	$\Delta_{eq}3$
			293.15 K			
0.2545	0.7679	1.2401	1.2399	1.2400	−0.0002	−0.0001
0.5027	0.5111	1.2272	1.2274	1.2276	0.0002	0.0004
0.7527	0.2523	1.2153	1.2148	1.2149	−0.0005	−0.0004
0.4944	1.4988	1.4265	1.4269	1.4270	0.0004	0.0005
0.9897	0.9998	1.4077	1.4075	1.4077	−0.0002	0.0000
1.4843	0.5014	1.3884	1.3882	1.3883	−0.0002	−0.0001
0.7655	2.3177	1.6003	1.5999	1.6000	−0.0004	−0.0003
1.4879	1.5503	1.5683	1.5681	1.5682	−0.0002	−0.0001
2.2402	0.7513	1.5355	1.5346	1.5347	−0.0009	−0.0008
0.9855	2.9766	1.7147	1.7142	1.7142	−0.0005	−0.0005
1.9734	1.9830	1.6823	1.6817	1.6816	−0.0006	−0.0007
2.9596	0.9912	1.6496	1.6490	1.6488	−0.0006	−0.0008
				δ_ρ^{eqi} [a]	$2.7\cdot10^{-4}$	$2.6\cdot10^{-4}$
			298.15 K			
0.2545	0.7679	1.2370	1.2373	1.2374	0.0003	0.0004
0.5027	0.5111	1.2245	1.2249	1.2250	0.0004	0.0005
0.7527	0.2523	1.2130	1.2123	1.2124	−0.0007	−0.0006
0.4944	1.4988	1.4235	1.4236	1.4237	0.0001	0.0002
0.9897	0.9998	1.4045	1.4042	1.4044	−0.0003	−0.0001
1.4843	0.5014	1.3853	1.3848	1.3849	−0.0005	−0.0004
0.7655	2.3177	1.5964	1.5959	1.5960	−0.0005	−0.0004
1.4879	1.5503	1.5643	1.5641	1.5643	−0.0002	0.0000
2.2402	0.7513	1.5315	1.5307	1.5308	−0.0008	−0.0007
0.9855	2.9766	1.7107	1.7101	1.7101	−0.0006	−0.0006
1.9734	1.9830	1.6781	1.6776	1.6774	−0.0005	−0.0007
2.9596	0.9912	1.6453	1.6448	1.6445	−0.0005	−0.0008
				δ_ρ^{eqi} [a]	$3.1\cdot10^{-4}$	$3.1\cdot10^{-4}$
			308.15 K			
0.2545	0.7679	1.2319	1.2316	1.2317	−0.0003	−0.0002
0.5027	0.5111	1.2189	1.2192	1.2194	0.0003	0.0005
0.7527	0.2523	1.2062	1.2068	1.2069	0.0006	0.0007
0.4944	1.4988	1.4162	1.4163	1.4164	0.0001	0.0002
0.9897	0.9998	1.3973	1.3970	1.3971	−0.0003	−0.0002
1.4843	0.5014	1.3781	1.3776	1.3777	−0.0005	−0.0004
0.7655	2.3177	1.5875	1.5873	1.5874	−0.0002	−0.0001
1.4879	1.5503	1.5562	1.5556	1.5557	−0.0006	−0.0005
2.2402	0.7513	1.5232	1.5223	1.5223	−0.0009	−0.0009
0.9855	2.9766	1.7015	1.7008	1.7008	−0.0007	−0.0007
1.9734	1.9830	1.6691	1.6683	1.6682	−0.0008	−0.0009
2.9596	0.9912	1.6363	1.6355	1.6353	−0.0008	−0.0010
				δ_ρ^{eqi} [a]	$3.4\cdot10^{-4}$	$3.6\cdot10^{-4}$

[a]

$$\delta_\rho^{eqi} = \sum_{j=1}^{N} (|\rho_{j(eqi)} - \rho_{j(exp)}|/\rho_{j(exp)})/N$$

of $\delta_\rho^{eq\ 3}$ at the three temperatures are $2.0\cdot10^{-4}$, $2.8\cdot10^{-4}$, and $2.4\cdot10^{-4}$, respectively. Further comparisons using the reported densities[25−28] at 298.15 K show that the average relative differences ($\delta_\rho^{eq\ 2}/\delta_\rho^{eq\ 3}$) between the predicted and measured densities are $3.5\cdot10^{-4}/3.5\cdot10^{-4}$ for NaCl + KCl + H_2O (I_{max} = 4.4 mol·kg^{-1}), $2.2\cdot10^{-4}/2.2\cdot10^{-4}$ for KCl + $CaCl_2$ + H_2O (I_{max} = 3.6 mol·kg^{-1}), $3.5\cdot10^{-4}/3.8\cdot10^{-4}$ for HCl + KCl + NaCl + H_2O (I_{max} = 2.2 mol·kg^{-1}), and $3.6\cdot10^{-4}/5.9\cdot10^{-4}$ for Na_2SO_4 + NaCl + H_2O (I_{max} = 1.9 mol·kg^{-1}), in which I_{max} is the maximum ionic strength. The above results indicate that eqs 2 and 3 hold well for the mixed electrolyte solutions. One of the advantages of the semi-ideal solution theory is that its simple equations are applicable to aqueous solutions of electrolyte mixtures, nonelectrolyte mixtures, and (electrolyte + nonelectrolyte) mixtures.[1,2,9] By comparison, the equation of Patwardhan and Kumar is not applicable to the mixtures involving nonelectrolyte solutes, as they are based on the ionic strength fraction, which is not defined for nonelectrolyte solutes.

Table 4. Comparisons of Measured and Predicted Densities of the Ternary Systems $Y(NO_3)_3$ (B) + $Nd(NO_3)_3$ (C) + H_2O (A) at Different Temperatures and with $I_{max} \leq 24.5$ mol·kg^{-1}

m_B		ρ (g·cm^{-3})			$\Delta\rho$ (g·cm^{-3})	
(mol·kg^{-1})	m_C	exp	eq 2	eq 3	$\Delta_{eq\,2}$	$\Delta_{eq\,3}$
			293.15 K			
0.2532	0.7618	1.2416	1.2417	1.2418	0.0001	0.0002
0.5005	0.5083	1.2282	1.2287	1.2287	0.0005	0.0005
0.7512	0.2514	1.2163	1.2155	1.2155	−0.0008	−0.0008
0.5132	1.5514	1.4477	1.4474	1.4475	−0.0003	−0.0002
1.0147	1.0217	1.4211	1.4207	1.4208	−0.0004	−0.0003
1.5034	0.5055	1.3949	1.3945	1.3945	−0.0004	−0.0004
0.7595	2.2906	1.6075	1.6067	1.6068	−0.0008	−0.0007
1.5011	1.5142	1.5723	1.5718	1.5719	−0.0005	−0.0004
2.2355	0.7454	1.5382	1.5370	1.5370	−0.0012	−0.0012
1.0125	3.0658	1.7442	1.7429	1.7430	−0.0013	−0.0012
2.0077	2.0254	1.7011	1.7002	1.7003	−0.0009	−0.0008
2.9764	1.0127	1.6591	1.6582	1.6582	−0.0009	−0.0009
				δ_ρ^{eqi} [a]	$4.4\cdot10^{-4}$	$4.2\cdot10^{-4}$
			298.15 K			
0.2532	0.7618	1.2391	1.2392	1.2392	0.0001	0.0001
0.5005	0.5083	1.2255	1.2262	1.2262	0.0007	0.0007
0.7512	0.2514	1.2137	1.2130	1.2131	−0.0007	−0.0006
0.5132	1.5514	1.4443	1.4439	1.4440	−0.0004	−0.0003
1.0147	1.0217	1.4178	1.4173	1.4173	−0.0005	−0.0005
1.5034	0.5055	1.3916	1.3911	1.3911	−0.0005	−0.0005
0.7595	2.2906	1.6029	1.6025	1.6026	−0.0004	−0.0003
1.5011	1.5142	1.5690	1.5677	1.5678	−0.0013	−0.0012
2.2355	0.7454	1.5339	1.5329	1.5330	−0.0010	−0.0009
1.0125	3.0658	1.7398	1.7385	1.7382	−0.0013	−0.0016
2.0077	2.0254	1.6970	1.6960	1.6956	−0.0010	−0.0014
2.9764	1.0127	1.6548	1.6541	1.6537	−0.0007	−0.0011
				δ_ρ^{eqi} [a]	$4.8\cdot10^{-4}$	$5.0\cdot10^{-4}$
			308.15 K			
0.2532	0.7618	1.2336	1.2334	1.2335	−0.0002	−0.0001
0.5005	0.5083	1.2202	1.2205	1.2206	0.0003	0.0004
0.7512	0.2514	1.2079	1.2075	1.2075	−0.0004	−0.0004
0.5132	1.5514	1.4368	1.4364	1.4365	−0.0004	−0.0003
1.0147	1.0217	1.4105	1.4099	1.4099	−0.0006	−0.0006
1.5034	0.5055	1.3840	1.3838	1.3838	−0.0002	−0.0002
0.7595	2.2906	1.5942	1.5936	1.5937	−0.0006	−0.0005
1.5011	1.5142	1.5599	1.5590	1.5591	−0.0009	−0.0008
2.2355	0.7454	1.5255	1.5244	1.5245	−0.0011	−0.0010
1.0125	3.0658	1.7298	1.7289	1.7286	−0.0009	−0.0012
2.0077	2.0254	1.6876	1.6865	1.6862	−0.0011	−0.0014
2.9764	1.0127	1.6460	1.6447	1.6444	−0.0013	−0.0016
				δ_ρ^{eqi} [a]	$4.4\cdot10^{-4}$	$4.6\cdot10^{-4}$

[a] $$\delta_\rho^{eqi} = \sum_{j=1}^{N} (|\rho_{j(eqi)} - \rho_{j(exp)}|/\rho_{j(exp)})/N$$

Table 5. Comparisons of Measured and Predicted Densities of the Ternary Systems $Ce(NO_3)_3$ (B) + $Nd(NO_3)_3$ (C) + H_2O (A) at Different Temperatures and with $I_{max} \leq 24.5$ mol·kg^{-1}

m_B		ρ (g·cm^{-3})			$\Delta\rho$ (g·cm^{-3})	
(mol·kg^{-1})	m_C	exp	eq 2	eq 3	$\Delta_{eq\,2}$	$\Delta_{eq\,3}$
			293.15 K			
0.2565	0.7673	1.2546	1.2545	1.2545	−0.0001	−0.0001
0.5097	0.5165	1.2534	1.2539	1.2539	0.0005	0.0005
0.7720	0.2568	1.2538	1.2534	1.2534	−0.0004	−0.0004
0.5182	1.5501	1.4672	1.4672	1.4672	0.0000	0.0000
1.0233	1.0207	1.4603	1.4601	1.4601	−0.0002	−0.0002
1.5159	0.5042	1.4528	1.4530	1.4530	0.0002	0.0002
0.7723	2.3246	1.6402	1.6399	1.6399	−0.0003	−0.0003
1.5687	1.5396	1.6377	1.6376	1.6377	−0.0001	0.0000
2.3415	0.7779	1.6362	1.6354	1.6354	−0.0008	−0.0008
1.0189	3.0651	1.7768	1.7758	1.7758	−0.0010	−0.0010
2.0174	2.0272	1.7659	1.7659	1.7659	0.0000	0.0000
3.0001	1.0057	1.7563	1.7561	1.7561	−0.0002	−0.0002
				δ_ρ^{eqi} [a]	$2.1\cdot10^{-4}$	$2.0\cdot10^{-4}$
			298.15 K			
0.2565	0.7673	1.2517	1.2519	1.2519	0.0002	0.0002
0.5097	0.5165	1.2518	1.2513	1.2513	−0.0005	−0.0005
0.7720	0.2568	1.2511	1.2507	1.2507	−0.0004	−0.0004
0.5182	1.5501	1.4641	1.4638	1.4638	−0.0003	−0.0003
1.0233	1.0207	1.4570	1.4567	1.4567	−0.0003	−0.0003
1.5159	0.5042	1.4496	1.4497	1.4497	0.0001	0.0001
0.7723	2.3246	1.6362	1.6357	1.6357	−0.0005	−0.0005
1.5687	1.5396	1.6342	1.6335	1.6335	−0.0007	−0.0007
2.3415	0.7779	1.6319	1.6313	1.6313	−0.0006	−0.0006
1.0189	3.0651	1.7722	1.7712	1.7712	−0.0010	−0.0010
2.0174	2.0272	1.7618	1.7614	1.7615	−0.0004	−0.0003
3.0001	1.0057	1.7521	1.7518	1.7519	−0.0003	−0.0002
				δ_ρ^{eqi} [a]	$2.9\cdot10^{-4}$	$2.8\cdot10^{-4}$
			308.15 K			
0.2565	0.7673	1.2457	1.2460	1.2461	0.0003	0.0004
0.5097	0.5165	1.2453	1.2455	1.2455	0.0002	0.0002
0.7720	0.2568	1.2445	1.2449	1.2449	0.0004	0.0004
0.5182	1.5501	1.4562	1.4562	1.4562	0.0000	0.0000
1.0233	1.0207	1.4495	1.4492	1.4492	−0.0003	−0.0003
1.5159	0.5042	1.4421	1.4423	1.4423	0.0002	0.0002
0.7723	2.3246	1.6276	1.6268	1.6268	−0.0008	−0.0008
1.5687	1.5396	1.6249	1.6247	1.6247	−0.0002	−0.0002
2.3415	0.7779	1.6235	1.6226	1.6226	−0.0009	−0.0009
1.0189	3.0651	1.7620	1.7616	1.7616	−0.0004	−0.0004
2.0174	2.0272	1.7522	1.7519	1.7520	−0.0003	−0.0002
3.0001	1.0057	1.7428	1.7424	1.7425	−0.0004	−0.0003
				δ_ρ^{eqi} [a]	$2.4\cdot10^{-4}$	$2.4\cdot10^{-4}$

[a] $$\delta_\rho^{eqi} = \sum_{j=1}^{N} (|\rho_{j(eqi)} - \rho_{j(exp)}|/\rho_{j(exp)})/N$$

The advantage the equation of Patwardhan and Kumar is that it does not require the osmotic coefficient data of the binary solutions. Equation 2 needs the osmotic coefficients of the binary solutions. However, as can be seen from the present results and the previous results,[29] the osmotic coefficients measured at 298.15 K can be used to determine the molalities $m_{M_iX_i}^{(io)}$ of the binary solutions at other temperatures, which can then be used to provide good predictions for the densities or the surface tensions of the mixed electrolyte solutions in the (293.15 to 308.15) K range and in the (283.15 to 343.15) K range, respectively. Because the osmotic coefficients of the binary electrolyte solutions at 298.15 K have been extensively reported in the literature, the use of the osmotic coefficients of the binary solutions makes the calculations more complicated but exerts little influence on the application of the corresponding equations.

Conclusions

Densities were measured for the ternary systems $Y(NO_3)_3$ + $Ce(NO_3)_3$ + H_2O, $Y(NO_3)_3$ + $Nd(NO_3)_3$ + H_2O, and $Ce(NO_3)_3$ + $Nd(NO_3)_3$ + H_2O and their binary subsystems at (293.15, 298.15, and 308.15) K and then were used to test the predictability of the simple density equations. The comparison results show that eqs 2 and 3 can yield good predictions for the densities of the ternary electrolyte solutions in terms of the properties of the binary solutions that do not involve mixed parameters, which indicates that these equations can make use of the information on the binary solutions, avoiding much complexity in the calculation of densities of the mixed solutions, and provide good predictions for the densities of the mixed solutions. The comparison results also show that the extensively reported osmotic coefficients of the binary solutions measured at 298.15 K can be used to determine the compositions of the binary solutions having the same water activity as that of the mixed solutions at other temperatures.

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